AP-C Angular Momentum

AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Calculating Angular Momentum
- a. Calculate the angular momentum vector for a moving particle.
- b. Calculate the angular momentum vector for a rotating rigid object where angular momentum is parallel to the angular velocity.

▼ 2. Conservation of Angular Momentum
- a. Recognize conditions under which angular momentum is conserved and relate this to systems such as satellite orbits.
- b. State the relation between net torque and angular momentum.
- c. Analyze problems in which the moment of inertia of an object is changed as it rotates freely about a fixed axis.
- d. Analyze a collision between a moving particle and a rigid object that can rotate.

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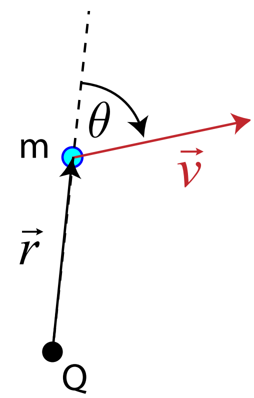
Momentum

Momentum (**p**) is a vector describing how difficult it is to stop a moving object.
Total momentum is the sum of individual momenta.
A mass with velocity **v** has momentum **p**=m**v**.



Angular Momentum

Angular momentum (**L**) is a vector describing how difficult it is to stop a rotating object.
Total angular momentum is the sum of individual angular momenta.
A mass with velocity **v** moving at some position **r** about point Q has angular momentum $\mathbf{L_Q}$
Note that angular momentum depends on your point of origin!

$$\vec{L}_Q = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = (\vec{r} \times \vec{v})m$$

$$\left|\vec{L}_Q\right| = mvr\sin\theta \xrightarrow{v=\omega r}$$

$$\left|\vec{L}_Q\right| = mr^2\omega$$

Spin Angular Momentum

For an object rotating about its center of mass, L=Iω. This is an intrinsic property of an object rotating about its center of mass, and is known as the object's **spin angular momentum**. It is constant even if you calculate it relative to any point in space.

Angular Momentum of an Object in Circular Orbit

Find the angular momentum of a planet orbiting the sun, assuming it follows a circular orbit.

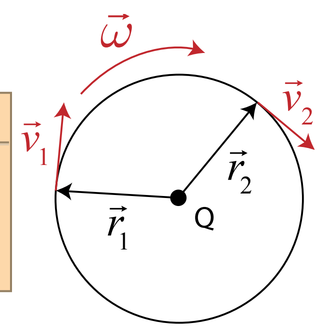
$$\vec{L}_Q = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = (\vec{r} \times \vec{v})m \rightarrow$$

$$\left|\vec{L}_Q\right| = m\left|\vec{r} \times \vec{v}\right| = mvr\sin\theta \xrightarrow{\theta=90°}$$

$$\left|\vec{L}_Q\right| = mvr$$

Note

Use RHR to determine direction of angular momentum vector (into the plane of the paper). The angular momentum is constant with respect to point Q.



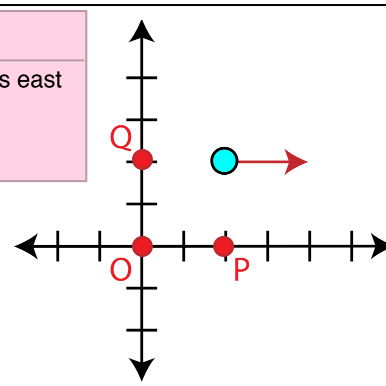
Angular Momentum of a Point Particle

Find the angular momentum for a 5 kg point particle located at (2,2) with a velocity of 2 m/s east
A) about the origin (point O)
B) about point P at (2,0)
C) about point Q at (0,2)

$$\left|\vec{L}_O\right| = mvr\sin\theta = (5)(2)(2\sqrt{2})\sin 45° = 20\ {}^{kg\bullet m^2}/_{s}$$

$$\left|\vec{L}_P\right| = mvr\sin\theta = (5)(2)(2)\sin 90° = 20\ {}^{kg\bullet m^2}/_{s}$$

$$\left|\vec{L}_Q\right| = mvr\sin\theta = (5)(2)(2)\sin 0° = 0$$



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1. Conservation of Angular Momentum
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Relationship Between Angular Momentum and Net Torque

Consider the angular momentum about point Q for an object with momentum **p**.

$$\vec{L}_Q = \vec{r} \times \vec{p} \xrightarrow{\frac{d}{dt}(\vec{A}\times\vec{B})=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}} \frac{d\vec{L}_Q}{dt} = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} \xrightarrow[\frac{d\vec{p}}{dt}=\vec{F}]{\frac{d\vec{r}}{dt}=\vec{v}}$$

$$\frac{d\vec{L}_Q}{dt} = \vec{v}\times\vec{p} + \vec{r}\times\vec{F} \xrightarrow{\vec{v}\text{ and }\vec{p}\text{ in same direction }\rightarrow \vec{v}\times\vec{p}=0} \frac{d\vec{L}_Q}{dt} = \vec{r}\times\vec{F} \xrightarrow{\vec{r}\times\vec{F}=\vec{\tau}}$$

$$\frac{d\vec{L}_Q}{dt} = \vec{\tau}_Q$$

Notes

A torque on an object changes its angular momentum.
A change in angular momentum is caused by a torque.

Conservation of Angular Momentum

Spin angular momentum, the product of an object's moment of inertia and its angular velocity about the center of mass, is conserved in a closed system with no external net torques applied.

$$\vec{L} = I\vec{\omega}$$

Ice Skater Sample Problem

An ice skater spins with a specific angular velocity. She brings her arms and legs closer to her body, reducing her moment of inertia to half its original value. What happens to her angular velocity? What happens to her rotational kinetic energy?

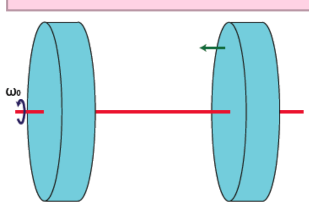
As the skater pulls her arms and legs in, she reduces her moment of inertia. Since there is no external net torque, her spin angular momentum remains constant, therefore her angular velocity must double. Rotational kinetic energy, on the other hand, is governed by $K=0.5I\omega^2$. Moment of inertia is cut in half, but angular velocity is doubled, therefore rotational kinetic energy is doubled. The skater must have done work in pulling her arms and legs in while spinning!

$$\vec{L} = I\vec{\omega}$$

$$K = \tfrac{1}{2}I\omega^2$$

Combining Spinning Discs

A disc with moment of inertia 1 kg·m² spins about an axle through its center of mass with angular velocity 10 rad/s. An identical disc which is not rotating is slid along the axle until it makes contact with the first disc. If the two discs stick together, what is their combined angular velocity?



$$L_0 = L \rightarrow I_0\omega_0 = I\omega \rightarrow$$

$$\omega = \frac{I_0\omega_0}{I} = \frac{(1kg \bullet m^2)(10\,{}^{rad}\!/\!{}_s)}{2kg \bullet m^2} = 5\,{}^{rad}\!/\!{}_s$$