AP-C Objectives (from College Board Learning Objectives for AP Physics)

▼ 1. Work

- a. Calculate the work done by a specified constant force on an object that undergoes a specified displacement.
- b. Relate the work done by a force to the area under a graph of force as a function of position, and calculate this work in the case where the force is a linear function of position.
- c. Use integration to calculate the work performed by a force **F**(**x**) on an object that undergoes a specified displacement in one dimension.
- d. Use the scalar product operation to calculate the work performed by a specified constant force F on an object that undergoes a displacement in a plane.

▼ 2. Work-Energy Theorem

- a. Calculate the change in kinetic energy or speed that results from performing a specified amount of work on an object.
- b. Calculate the work performed by the net force, or by each of the forces that make up the net force, on an object that undergoes a specified change in speed or kinetic energy.
- c. Apply the theorem to determine the change in an object's kinetic energy and speed that results from the application of specified forces, or to determine the force that is required in order to bring an object to rest in a specified distance.

▼ 3. Conservative Forces

- a. State alternative definitions of "conservative force" and explain why these definitions are equivalent.
- b. Describe examples of conservative forces and non-conservative forces.

▼ 4. Potential Energy

- a. State the general relation between force and potential energy, and explain why potential energy can be associated only with conservative forces.
- b. Calculate a potential energy function associated with a specified one-dimensional force **F**(**x**).
- c. Calculate the magnitude and direction of a one-dimensional force when given the potential energy function **U**(**x**) for the force.
- d. Write an expression for the force exerted by an ideal spring and for the potential energy of a stretched or compressed spring.
- e. Calculate the potential energy of one or more objects in a uniform gravitational field.

▼ 5. Conservation of Energy

- a. State and apply the relation between the work performed on an object by nonconservative forces and the change in an object's mechanical energy.
- b. Describe and identify situations in which mechanical energy is converted to other forms of energy.
- c. Analyze situations in which an object's mechanical energy is changed by friction or by a specified externally applied force.
- d. Identify situations in which mechanical energy is or is not conserved.
- e. Apply conservation of energy in analyzing the motion of systems of connected objects, such as an Atwood's machine.
- f. Apply conservation of energy in analyzing the motion of objects that move under the influence of springs.
- g. Apply conservation of energy in analyzing the motion of objects that move under the influence of other non-constant one-dimensional forces.
- h. Students should be able to recognize and solve problems that call for application both of conservation of energy and Newton's Laws.

▼ 6. Power

- a. Calculate the power required to maintain the motion of an object with constant acceleration (e.g., to move an object along a level surface, to raise an object at a constant rate, or to overcome friction for an object that is moving at a constant speed).
- b. Calculate the work performed by a force that supplies constant power, or the average power supplied by a force that performs a specified amount of work.

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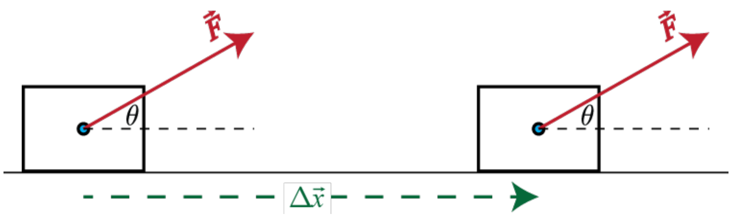
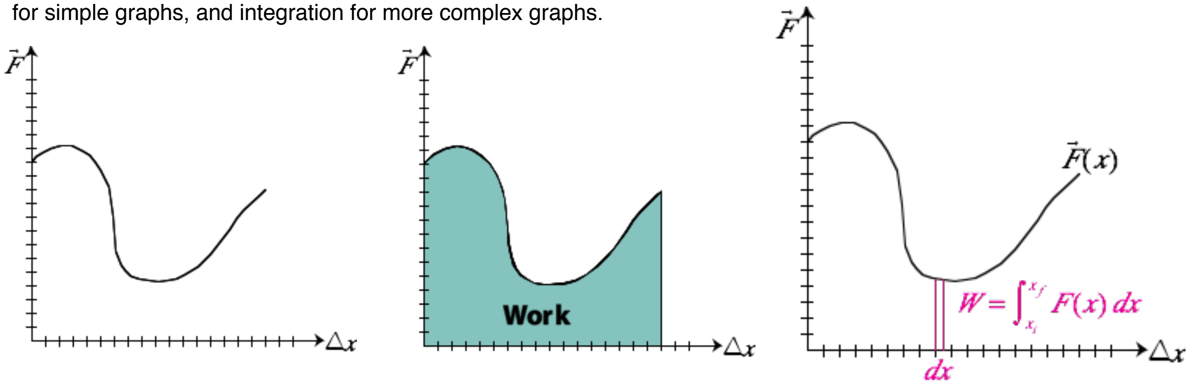
When a force acts on an object to cause a displacement, the force has done **work** on the object. For a constant force:

$$W = F\cos\theta\Delta x = \vec{F}\bullet\Delta\vec{x}$$

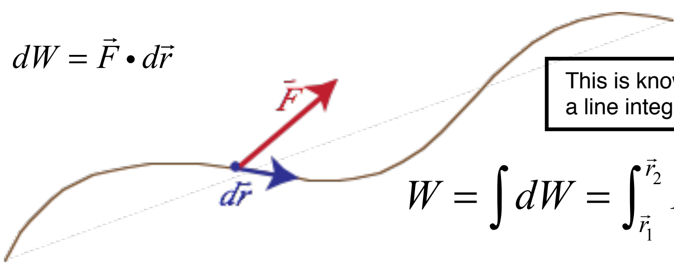


For a non-constant force, the work done is the area under the force vs. displacement graph. You can use geometry for simple graphs, and integration for more complex graphs.



In more than one dimension, you have to add up all the little bits of work done by the force for each little displacement.

$$dW = \vec{F}\bullet d\vec{r}$$



This is known as a line integral

$$W = \int dW = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\bullet d\vec{r}$$

▼ General Form

$$W = \int \vec{F}\bullet d\vec{r}$$

▼ Example: Energy Stores in a Spring

A spring obeys Hooke's Law, where F(x)=-kx. How much work is done in compressing a spring from equilibrium to some point x?

$$W = \int \vec{F}\bullet d\vec{r} \xrightarrow{d\vec{r}=d\vec{x}} W = \int \vec{F}\bullet d\vec{x} \rightarrow W = \int_0^x kx\,dx \rightarrow W = k\int_0^x x\,dx = k\left.\frac{x^2}{2}\right|_0^x = \tfrac{1}{2}kx^2$$

This is the potential energy now stored in the spring, since exactly that much work was done to compress the spring.

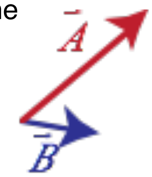
▼ Dot Product (aka Scalar Product)

The dot product is a mathematical operation which takes two vectors and provides a scalar product.

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z + \ldots$$

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta = AB\cos\theta$$

The dot product of perpendicular vectors is zero. You can think of the dot product as the length of the projection of vector **A** onto the unit vector of **B**.



▼ Example: Find the Dot Product

If vector **A** = <-7,4> and vector **B** = <-2,9>, what is **A** • **B**?

A • **B**= –7×-2 + 4×9 = 50

What is the angle between **A** and **B**?

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta \rightarrow (\sqrt{65})(\sqrt{85})\cos\theta = 50 \rightarrow \theta = 47.7°$$

▼ Example: Find Component for Perpendicular Vectors

If **A** = <-2,4> and **B** = <6,B_y>, find the value of B_y such that **A** and **B** are perpendicular.

Hint: Two vectors are perpendicular if their dot product is zero.

$$\vec{A} \bullet \vec{B} = 0 \rightarrow < -2,4 > \bullet < 6, B_y > = 0 \rightarrow (-2 \times 6) + (4 \times B_y) = 0 \rightarrow$$

$$4B_y - 12 = 0 \rightarrow 4B_y = 12 \rightarrow B_y = 3$$

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When you do work on an object, you give it energy. When an object transfers energy to another object, it does work.

▼ Derivation of the Work-Energy Theorem in 1-D

$$W_{AB} = \int_A^B \vec{F} \bullet d\vec{r} \xrightarrow{1-D} W = \int_A^B F_x \, dx \xrightarrow[dx=vdt]{F=ma=m\frac{dv}{dt}} W = \int_{v_A}^{v_B} mv \, dv \rightarrow$$

$$W = \left. \tfrac{mv^2}{2} \right|_{v_A}^{v_B} = \tfrac{1}{2} m(v_B^2 - v_A^2) = \Delta KE$$

▼ Example: Force Required to Stop a Train

A train car with a mass of 200kg is traveling at 20 m/s. How much force must the brakes exert in order to stop the train car in a distance of 10 meters?

$$W_{net} = \Delta KE = KE_f - KE_i = 0 - \tfrac{1}{2}(200kg)(20 \tfrac{m}{s})^2 = -40{,}000J$$

$$W_{net} = F \times d \rightarrow F = \frac{W_{net}}{d} = \frac{-40{,}000J}{10m} = -4{,}000N$$

You can verify this using kinematics and Newton's 2nd Law

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  - a. State alternative definitions of "conservative force" and explain why these definitions are equivalent.
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Conservative Force Definitions

A force in which the work done on an object is **independent of the path** taken is known as a conservative force.

A force in which the **work done moving along a closed path is zero.**

A force in which the work done is directly related to a negative change in potential energy. ($W = -\Delta U$)

Non-Conservative Force Definitions

A force in which the work done on an object is **dependent upon the path** taken is known as a non-conservative force.

A force in which the **work done moving along a closed path is not zero.**

A force in which the work done is not necessarily related to a negative change in potential energy.

Conservative Forces	Non-Conservative Forces
Gravity	Friction
Elastic	Drag
Coulombic	Air Resistance

Potential Energy

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   - d. Write an expression for the force exerted by an ideal spring and for the potential energy of a stretched or compressed spring.
   - e. Calculate the potential energy of one or more objects in a uniform gravitational field.

Potential energy is energy an object possesses due to its position or condition. Potential Energy is often symbolized as **PE** or **U**.

Common Potential Energy Formulas

$\Delta U_g = mgh$ In uniform gravitational field

$U_s = \frac{1}{2}kx^2$

Work Done By A Conservative Force

$$W = -\Delta U$$

$$\Delta U = -W_{cons.F} = -\int_{x_1}^{x_2} \vec{F} \bullet d\vec{x}$$

Gravitational Potential Energy in a Non-Uniform Gravitational Field

Start with Newton's Law of Universal Gravitation $F_g = -\dfrac{Gm_1m_2}{r^2}$

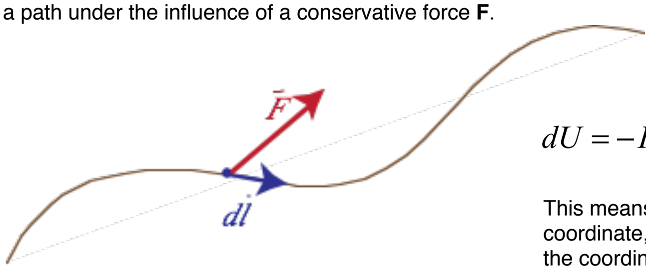
Then:

$$\Delta U = -W_{consF} = -W_{grav} = -\int_{\infty}^{r} \frac{-Gm_1m_2}{r^2}dr \rightarrow U_g = Gm_1m_2 \int_{\infty}^{r} \frac{dr}{r^2} \rightarrow$$

$$U_g = Gm_1m_2\left(\frac{1}{r}\bigg|_{\infty}^{r}\right) = \frac{-Gm_1m_2}{r}$$

Force from Potential Energy

Imagine a small displacement **dl** of an object moving along a path under the influence of a conservative force **F**.



The change in the potential energy is:

$$dU = -dW_F = -\vec{F} \bullet d\vec{l} = -F\cos\theta dl$$

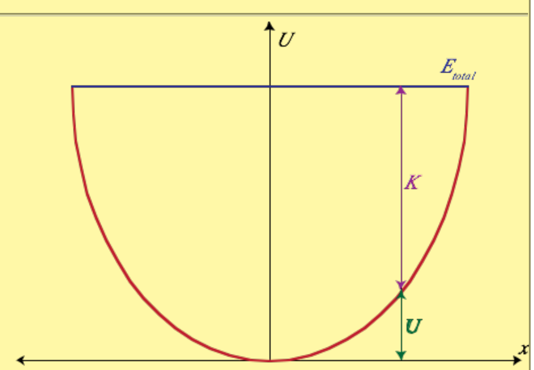
Fcosθ is the component of **F** in the direction of **dl**, which can be written as $F\cos\theta = F_l$. Therefore:

$$dU = -F\cos\theta dl \xrightarrow{F\cos\theta = F_l} dU = -F_l dl \rightarrow F_l = -\frac{dU}{dl}$$

This means that if you know how the potential energy depends on a coordinate, you can find the component of the force in the direction of the coordinate.

Example: Derivation of Hooke's Law

Consider the graph of a spring's potential energy:
$U(x) = \frac{1}{2}kx^2$



Total Energy = E_{total}=K+U

Force at any given point on the curve is the opposite of the slope.

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$ ← Derivation of Hooke's Law!

Conservation of Energy

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- h. Students should be able to recognize and solve problems that call for application both of conservation of energy and Newton's Laws.

Consider a single conservative force doing work on a closed system:

$W_F = \Delta K$ Work-Energy Theorem

$W_F = -\Delta U$ Conservative Force

$$W_F = \Delta K = -\Delta U \rightarrow \Delta K + \Delta U = 0$$

This means that the sum of the kinetic and potential energies doesn't change. This quantity is called the **total mechanical energy (E)**. $E = K + U$

If only conservative forces are doing work, $E_i = E_f$, therefore $K_i + U_i = K_f + U_f$. This is the **Law of Conservation of Mechanical Energy**.

Non-conservative forces change the total mechanical energy of a system, but not the total energy of a system (work done by a non-conservative force is typically converted to internal energy (heat).

$$E_{total} = K + U + W_{nc}$$

$$E_{mech} = K + U$$

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▼ Power

Power is the rate at which work is done / the rate at which a force does work.

$$P_{avg}=\frac{\Delta W}{\Delta t}$$

Units of power are J/s, or Watts.

For instantaneous power, look at average power over a very small time interval

$$P=\frac{dW}{dt}\xrightarrow{dW=\vec{F}\bullet d\vec{r}}P=\frac{\vec{F}\bullet d\vec{r}}{dt}\xrightarrow{\vec{v}=\frac{d\vec{r}}{dt}}P=\vec{F}\bullet\vec{v}$$

$$P_{avg}=\frac{\Delta W}{\Delta t}$$

$$P=\frac{dW}{dt}$$

$$P=\vec{F}\bullet\vec{v}$$

Note: 1 horsepower = 746 Watts

▼ Example: Power Delivered to a Moving Mass

Find the power delivered by the net force to a 10-kg mass at t=4s given the position of the mass is given by $x(t)=4t^3-2t$.

First, find v(t) and a(t):

$v(t)=12t^2-2$

$a(t)=24t$

Next, find the net force using Newton's 2nd Law:

$F_{net}=ma=(10kg)(24t)=240t$

Then, find the power delivered:

$$P=\vec{F}\bullet\vec{v}=Fv\cos\theta\xrightarrow[\cos\theta=1]{\theta=0^\circ}P=(240t)(12t^2-2)=2880t^3-480t\xrightarrow{t=4s}$$

$$P=(2880)(4^3)-480(4)=182{,}400\ Watts$$