AP-C Objectives (from College Board Learning Objectives for AP Physics)

1. Charge and Coulomb’s Law
   - a. Describe the type of charge and the attraction and repulsion of charges
   - b. Describe polarization and induced charges.
   - c. Calculate the magnitude and direction of the force on a positive or negative charge due to other specified point charges.
   - d. Analyze the motion of a particle of specified charge and mass under the influence of an electrostatic force.
   - e. Describe the process of charging by induction.
   - f. Explain why a neutral conductor is attracted to a charged object.

2. Electric Field due to Point Charges
   - a. Define the electric field in terms of force on a test charge.
   - b. Describe and calculate the electric field produced by one or more point charges.
   - c. Calculate the magnitude and direction of the force on a positive or negative charge placed in a specified field.
   - d. Interpret electric field diagrams.

3. Gauss’s Law
   - a. Understand the relationship between electric field and electric flux.
      - i. Calculate the flux of an electric field through an arbitrary surface or of a uniform field over and perpendicular to a Gaussian surface.
      - ii. Calculate the flux of the electric field through a rectangle when the field is perpendicular to the rectangle and a function of one coordinate only.
      - iii. State and apply the relationship between flux and lines of force.
   - b. Understand Gauss’s Law
      - i. State Gauss’s Law in integral form and apply it qualitatively to relate flux and electric charge for a specified surface.
      - ii. Apply Gauss’s Law, along with symmetry arguments, to determine the electric field for a planar, spherical, or cylindrically symmetric charge distribution.
      - iii. Apply Gauss’s Law to determine the charge density or total charge on a surface in terms of the electric field near the surface.

4. Electric Fields due to Other Charge Distributions
   - a. Calculate the electric field of a straight, uniformly charged wire; the axis of a thin ring of charge; and the center of a circular arc of charge.
   - b. Identify situations in which the direction of the electric field produced by a charge distribution can be deduced from symmetry considerations.
   - c. Describe the patterns and variation with distance of the electric field of oppositely charged parallel plates; a long uniformly charged wire; a thin cylindrical shell; and a thin spherical shell.
   - d. Determine the fields of parallel charged planes, coaxial cylinders, and concentric spheres.
**Electric Field (E)**

The electric field describes the amount of electrostatic force observed by a charge placed at a point in the field per unit charge. The electric field vector points in the direction a positive test charge would feel a force. Electric field strength is measured in N/C, which are equivalent to V/m.

\[ E = \frac{F}{q} \]

\[ F = qE = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \]

Electric field lines indicate the direction of the electric force on a positive test charge.

**Electric Field due to Multiple Point Charges**

The electric field follows the law of superposition. In order to determine the electric field due to multiple point charges, add up the electric field produced by one or more point charges.

**Electric Force and Coulomb's Law**

Like charges repel, opposite charges attract.

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2 \]

**Conduction and Induction**

Charging by contact is known as conduction. If a charged conductor is brought into contact with an identical neutral conductor, the net charge will be shared across the two conductors. Charging an object without placing it in contact with another charged object is known as induction.

**Polarization and Electric Dipole Moment**

When a charged object is brought near a conductor, the electrons in the conductor are free to move. When a charged object is brought near an insulator, the electrons are not free to move, but they may spend a little more time on one side of their orbit than another, creating a net separation of charge in a process known as polarization. The distance between the shifted positive and negative charges, multiplied by the charge, is known as the electric dipole moment.

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- **2. Electric Field due to Point Charges**
  - a. Define the electric field in terms of force on a test charge
  - b. Describe and calculate the electric field produced by one or more point charges
  - c. Calculate the magnitude and direction of the force on a positive or negative charge placed in a specified field
  - d. Interpret electric field diagrams

**Sample Problem: E Field due to Point Charges**

Find the electric field at the origin due to the three point charges shown.

**Sample Problem: Where is the E Field Zero?**

Determine the x-coordinate where the electric field is zero using the diagram.

**Conductors and Insulators**

Charges can move freely in conductors. Charges cannot move freely in insulators.

**Atomic Particles**

- Protons have a charge of +1e.
- Electrons have a charge of -1e.
- Neutrons are neutral.
- Atoms with an excess of protons or electrons are known as ions.

**Electric Charge**

Electric charge (q) is a fundamental property of certain particles. The smallest amount of isolable charge is the elementary charge (e), equal to 1.6×10⁻¹⁹ coulombs. Charge can be positive or negative.

**Charging and Polarization**

- Charging by contact is known as conduction.
- Charging an object without placing it in contact with another charged object is known as induction.

**Polarization**

- When a charged object is brought near a conductor, the electrons in the conductor are free to move.
- When a charged object is brought near an insulator, the electrons are not free to move, but they may spend a little more time on one side of their orbit than another, creating a net separation of charge in a process known as polarization.

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**Polarization and Electric Dipole Moment**

When a charged object is brought near a conductor, the electrons in the conductor are free to move. When a charged object is brought near an insulator, the electrons are not free to move, but they may spend a little more time on one side of their orbit than another, creating a net separation of charge in a process known as polarization. The distance between the shifted positive and negative charges, multiplied by the charge, is known as the electric dipole moment.
**AP-C Objectives (from College Board Learning Objectives for AP Physics)**

- 1. Electric Fields due to Continuous Charge Distributions
  - a. Calculate the electric field of a straight, uniformly charged wire; the axis of a thin ring of charge; and the center of a circular arc of charge.
  - b. Identify situations in which the direction of the electric field produced by a charge distribution can be deduced from symmetry considerations.

**Electric Fields due to Other Charge Distributions 1**

**Symmetry Arguments**

- Horizontal component will cancel out since the charge is uniformly distributed, so we only need to worry about the vertical component of the electric field.

\[
\begin{align*}
\Delta E_x &= \frac{\lambda}{4\pi \varepsilon_0} \int_{0}^{\pi \over 2} \sin \theta \, d\theta \rightarrow E_x = \frac{\lambda}{2\pi \varepsilon_0 R} \\
\Delta E_y &= \frac{\lambda}{2\pi \varepsilon_0 R} \\
E &= \frac{\lambda}{2\pi \varepsilon_0 R}
\end{align*}
\]

**Strategy**

1. Divide the total charge \( Q \) into smaller charges \( \Delta Q \).
2. Find the electric field due to each \( \Delta Q \).
3. Find the total electric field by adding up the individual electric fields due to each \( \Delta Q \).
4. Realize the y-component of the electric field is 0 due to symmetry arguments.

\[
\begin{align*}
E_i &= E_i \cos \theta_i = \frac{1}{4\pi \varepsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i \left( \frac{r_i - \sqrt{x^2 + y^2}}{d} \right) \\
E_y &= \frac{\lambda}{4\pi \varepsilon_0} \int_{y}^{\pi \over 2} \sin \theta \, d\theta \rightarrow E_y = \frac{\lambda}{2\pi \varepsilon_0 R} \\
E_x &= \frac{\lambda}{2\pi \varepsilon_0 R} \\
E &= \frac{\lambda}{2\pi \varepsilon_0 R}
\end{align*}
\]

**What is the E Field if the rod is infinite?**

\[
E_x = \lim_{L \to \infty} \left( \frac{Q}{4\pi \varepsilon_0 d \left( \frac{x}{2} \right)^2 + d^2} \right) = \frac{Q}{2\pi \varepsilon_0 d L} \quad \text{acts like the E-field of a point charge!}
\]

Note that \( Q \) approaches infinity as \( L \) approaches infinity.
By symmetry, the only net electric field will be in the z-direction.

\[ E_z = \frac{\sigma}{2\pi \varepsilon_0} \left( \frac{z}{\sqrt{z^2 + R^2}} \right) \]

Find the electric field on the axis of a thin ring of charge.

\[ E_z = \frac{\lambda R}{2\pi \varepsilon_0} \]

\[ \Delta Q = \lambda R d\phi \]

Find the electric field due to a uniformly charged disk.

\[ E_z = \frac{\sigma}{2\pi \varepsilon_0} \]

\[ \Delta Q = \sigma \Delta A = \sigma 2\pi r \Delta r \]

Electric Field due to a Uniformly Charged Disk

\[ E_z = \frac{\sigma}{2\pi \varepsilon_0} \]

\[ \Delta Q = \sigma \Delta A = \sigma 2\pi r \Delta r \]

What is the electric field if the disc is infinite (an infinite plane)?

\[ \lim_{R \to \infty} \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{z}{\sqrt{z^2 + R^2}} \right) = \frac{\sigma}{2\varepsilon_0} \]

Electric Field due to a Finite Charged Rod

\[ \lambda = \frac{Q}{L} \rightarrow \Delta Q = \lambda \Delta x \rightarrow dQ = \lambda dx \]

\[ E_z = \frac{\lambda}{4\pi \varepsilon_0} \]
Consider a point charge inside a spherical shell of radius $R$. Determine the flux through the sphere.

**Electric Flux ($\Phi$)** is the amount of electric field penetrating a surface.

By symmetry, the electric field at all points on the cylinder must point radially in or out.

Outside the shell of charge ($r > R$):

$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

Inside the shell of charge ($r < R$):

Electric Flux Through Open Surfaces

$$\Phi = \oint d\Phi = \oint E \cdot dA$$

Choose a “Gaussian Surface” as a cylinder as shown in the diagram. By symmetry, the electric field at all points on the cylinder must point perpendicular to the plane through the caps of the cylinder.

**Integral over closed surface**

$$\Phi = \oint E \cdot dA = \frac{Q_{\text{outside}}}{\varepsilon_0}$$

Gauss’s Law

Useful for finding the electric field due to charge distributions for cases of:

1. Spherical symmetry
2. Cylindrical symmetry
3. Planar symmetry

$$\Phi = \oint E \cdot dA = \frac{Q_{\text{within}}}{\varepsilon_0}$$

Consider a point charge inside a spherical shell of radius $R$. Determine the flux through the sphere.

$$d\Phi = E \cdot dA = EdA\cos\theta \rightarrow \Phi = \int d\Phi = \int E \cdot dA$$

$$\Phi = \oint E \cdot dA = \frac{Q_{\text{outside}}}{\varepsilon_0}$$

Gauss’s Law!

**Sample Problem: Electric Field due to a Thin Hollow Shell**

Consider a thin hollow shell of uniformly distributed charge $Q$. Find the electric field inside and outside the sphere.

Inside the shell of charge ($r = R$):

$$\oint E \cdot dA = \frac{Q_{\text{outside}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{outside}}}{4\pi \varepsilon_0 R^2}$$

Outside the shell of charge ($r > R$):

$$\oint E \cdot dA = \frac{Q_{\text{outside}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{outside}}}{4\pi \varepsilon_0 R^2}$$

Same answer as if all the charge $Q$ was placed at a point in the center of the sphere.

$$E = \frac{\sigma R}{2\varepsilon_0}$$

Note: There is NO dependence on distance from the plane!