

AP-C Objectives (from College Board Learning Objectives for AP Physics)

- ▼ 1. Charge and Coulomb's Law
 - a. Describe the type of charge and the attraction and repulsion of charges
 - b. Describe polarization and induced charges.
 - c. Calculate the magnitude and direction of the force on a positive or negative charge due to other specified point charges.
 - d. Analyze the motion of a particle of specified charge and mass under the influence of an electrostatic force.
 - e. Describe the process of charging by induction.
 - f. Explain why a neutral conductor is attracted to a charged object.
- ▼ 2. Electric Field due to Point Charges
 - a. Define the electric field in terms of force on a test charge.
 - b. Describe and calculate the electric field produced by one or more point charges.
 - c. Calculate the magnitude and direction of the force on a positive or negative charge placed in a specified field.
 - d. Interpret electric field diagrams.
- ▼ 3. Gauss's Law
 - ▼ a. Understand the relationship between electric field and electric flux.
 - i. Calculate the flux of an electric field through an arbitrary surface or of a uniform field over and perpendicular to a Gaussian surface.
 - ii. Calculate the flux of the electric field through a rectangle when the field is perpendicular to the rectangle and a function of one coordinate only.
 - iii. State and apply the relationship between flux and lines of force.
 - ▼ b. Understand Gauss's Law
 - i. State Gauss's Law in integral form and apply it qualitatively to relate flux and electric charge for a specified surface.
 - ii. Apply Gauss's Law, along with symmetry arguments, to determine the electric field for a planar, spherical, or cylindrically symmetric charge distribution.
 - iii. Apply Gauss's Law to determine the charge density or total charge on a surface in terms of the electric field near the surface.
- ▼ 4. Electric Fields due to Other Charge Distributions
 - a. Calculate the electric field of a straight, uniformly charged wire; the axis of a thin ring of charge; and the center of a circular arc of charge.
 - b. Identify situations in which the direction of the electric field produced by a charge distribution can be deduced from symmetry considerations.
 - c. Describe the patterns and variation with distance of the electric field of oppositely charged parallel plates; a long uniformly charged wire; a thin cylindrical shell; and a thin spherical shell.
 - d. Determine the fields of parallel charged planes, coaxial cylinders, and concentric spheres.



Charge and Coulomb's Law

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 - d. Interpret electric field diagrams.

▼ Electric Charge

Electric charge (q) is a fundamental property of certain particles. The smallest amount of isolatable charge is the elementary charge (e), equal to 1.6×10^{-19} coulombs. Charge can be positive or negative.

▼ Atomic Particles

protons have a charge of $+1e$
 electrons have a charge of $-1e$
 neutrons are neutral.
 Atoms with an excess of protons or electrons are known as ions.

▼ Conductors and Insulators

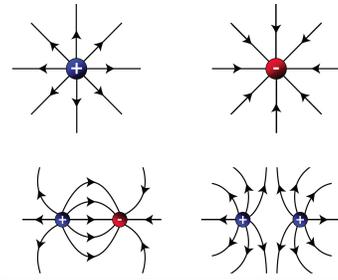
Charges can move freely in conductors.
 Charges cannot move freely in insulators.

▼ Electric Field (E)

The electric field describes the amount of electrostatic force observed by a charge placed at a point in the field per unit charge. The electric field vector points in the direction a positive test charge would feel a force. Electric field strength is measured in N/C, which are equivalent to V/m.

$$E = \frac{F}{q} \quad F = qE \quad E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Electric field lines indicate the direction of the electric force on a positive test charge.



▼ Electric Field due to Multiple Point Charges

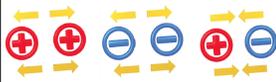
The electric field follows the law of superposition. In order to determine the electric field due to multiple point charges, add up the electric field due to each of the individual charges.



▼ Electric Force and Coulomb's Law

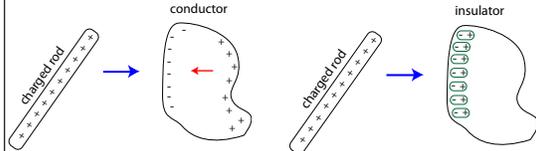
Like charges repel, opposite charges attract.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$



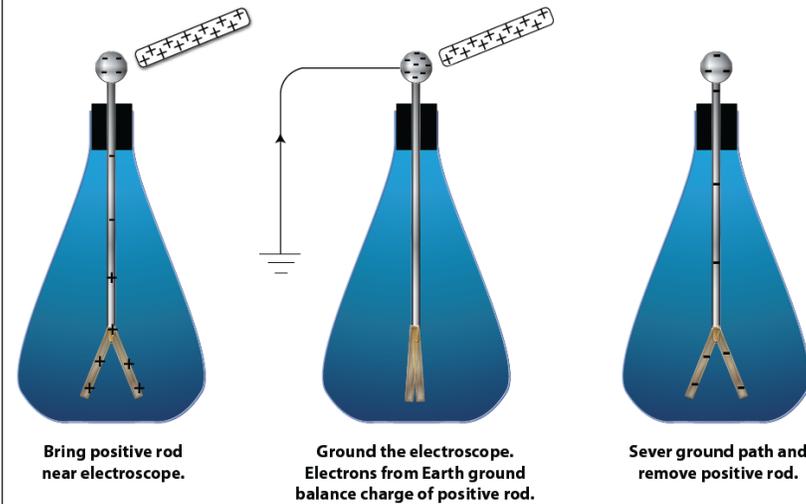
▼ Polarization and Electric Dipole Moment

When a charged object is brought near a conductor, the electrons in the conductor are free to move. When a charged object is brought near an insulator, the electrons are not free to move, but they may spend a little more time on one side of their orbit than another, creating a net separation of charge in a process known as **polarization**. The distance between the shifted positive and negative charges, multiplied by the charge, is known as the **electric dipole moment**.



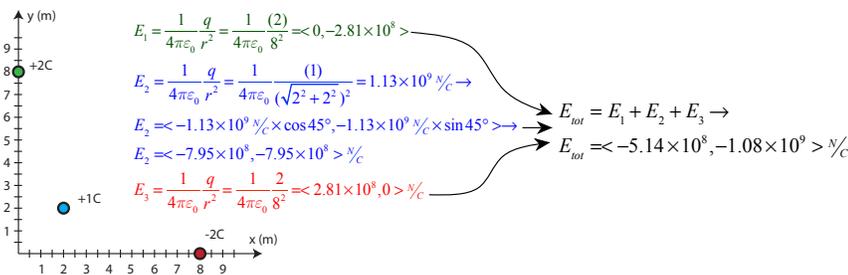
▼ Conduction and Induction

Charging by contact is known as conduction. If a charged conductor is brought into contact with an identical neutral conductor, the net charge will be shared across the two conductors. Charging an object without placing it in contact with another charged object is known as induction.



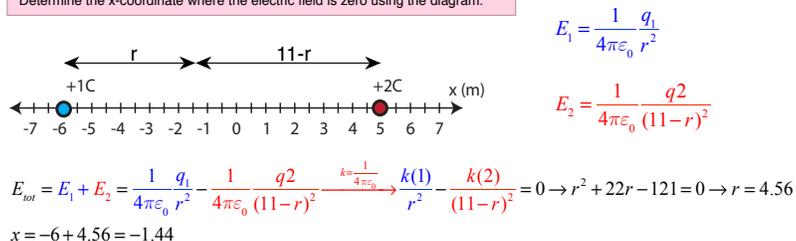
▼ Sample Problem: E Field due to Point Charges

Find the electric field at the origin due to the three point charges shown.



▼ Sample Problem: Where is the E Field Zero?

Determine the x-coordinate where the electric field is zero using the diagram.



Electric Fields due to Other Charge Distributions 1

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▼ 1. Electric Fields due to Continuous Charge Distributions

- a. Calculate the electric field of a straight, uniformly charged wire; the axis of a thin ring of charge; and the center of a circular arc of charge.
- b. Identify situations in which the direction of the electric field produced by a charge distribution can be deduced from symmetry considerations.

▼ Charge Densities

$$\text{Linear Charge Density } \lambda = \frac{\Delta Q}{\Delta L}$$

$$\text{Surface Charge Density } \sigma = \frac{\Delta Q}{\Delta A}$$

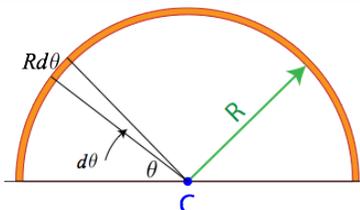
$$\text{Volume Charge Density } \rho = \frac{\Delta Q}{\Delta V}$$

▼ E-Field Due to a Thin Uniform Semicircle of Charge

A thin insulating semicircle of charge Q with radius R is centered around point C . Determine the electric field at point C due to the semicircle of charge.

▼ Symmetry Arguments

Horizontal component will cancel out since the charge is uniformly distributed, so we only need to worry about the vertical component of the electric field.



$$d\vec{E}_y = \frac{1}{4\pi\epsilon_0} \frac{dQ}{R^2} \sin\theta \xrightarrow{dQ=\lambda R d\theta} \int d\vec{E}_y = \int_{\theta=0}^{\theta=\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\theta}{R^2} \sin\theta d\theta \rightarrow \lambda = \frac{\Delta Q}{\Delta L} = \frac{Q}{\pi R}$$

$$\vec{E}_y = \frac{\lambda}{4\pi\epsilon_0 R} \int_{\theta=0}^{\theta=\pi} \sin\theta d\theta \rightarrow \vec{E}_y = \frac{\lambda}{4\pi\epsilon_0 R} (-\cos\theta) \Big|_0^\pi = \frac{2\lambda}{4\pi\epsilon_0 R} = \frac{\lambda}{2\pi\epsilon_0 R}$$

▼ E Field Due to a Thin Straight Insulating Wire

Find the electric field some distance d from a long straight insulating rod of length L at a point P which is perpendicular to the wire and equidistant from each end of the wire.

▼ Strategy

1. Divide the total charge Q into smaller charges ΔQ .
2. Find the electric field due to each ΔQ .
3. Find the total electric field by adding up the individual electric fields due to each ΔQ .
4. Realize the y -component of the electric field is 0 due to symmetry arguments.

$$E_{i_x} = E_i \cos\theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos\theta_i \xrightarrow{\substack{r_i = \sqrt{y_i^2 + d^2} \\ \cos\theta_i = \frac{d}{r_i} = \frac{d}{\sqrt{y_i^2 + d^2}}}} \rightarrow$$

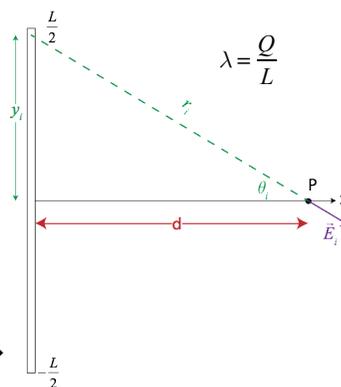
$$E_{i_x} = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{(y_i^2 + d^2)} \frac{d}{\sqrt{y_i^2 + d^2}} = \frac{1}{4\pi\epsilon_0} \frac{d \Delta Q}{(y_i^2 + d^2)^{3/2}} \xrightarrow{\Delta Q = \frac{Q}{L} dy}$$

$$E_{i_x} = \frac{1}{4\pi\epsilon_0} \frac{d Q dy}{L (y_i^2 + d^2)^{3/2}} \rightarrow E_x = \int_{y=-L/2}^{y=L/2} \frac{d Q / L}{4\pi\epsilon_0 (y^2 + d^2)^{3/2}} dy \rightarrow$$

$$E_x = \frac{d Q / L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dy}{(y^2 + d^2)^{3/2}} \xrightarrow{\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}} + C}$$

$$E_x = \frac{d Q / L}{4\pi\epsilon_0} \left[\frac{y}{d^2 \sqrt{y^2 + d^2}} \right]_{-L/2}^{L/2} = \frac{Q / L}{4\pi\epsilon_0 d} \left[\frac{L/2}{((L/2)^2 + d^2)^{1/2}} - \frac{-L/2}{((-L/2)^2 + d^2)^{1/2}} \right] \rightarrow$$

$$E_x = \frac{Q / L}{4\pi\epsilon_0 d} \frac{L}{((L/2)^2 + d^2)^{1/2}} = \frac{Q}{4\pi\epsilon_0 d \sqrt{(L/2)^2 + d^2}}$$



▼ What is the E Field if the rod is infinite?

$$E_x = \lim_{L \rightarrow \infty} \left(\frac{Q}{4\pi\epsilon_0 d \sqrt{(L/2)^2 + d^2}} \right) = \frac{Q}{4\pi\epsilon_0 d (L/2)} = \frac{Q}{2\pi\epsilon_0 d L} \xrightarrow{\lambda = \frac{Q}{L}} E_x = \frac{\lambda}{2\pi\epsilon_0 d}$$

Note that Q approaches infinity as L approaches infinity!

▼ What is the E field if the distance d is infinite?

$$E_x = \lim_{d \rightarrow \infty} \left(\frac{Q}{4\pi\epsilon_0 d \sqrt{(L/2)^2 + d^2}} \right) = \frac{Q}{4\pi\epsilon_0 d^2} \quad \text{Acts like the E-field of a point charge!}$$

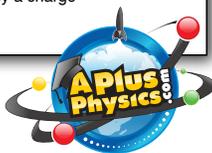


Electric Fields due to Other Charge Distributions 2

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- a. Calculate the electric field of a straight, uniformly charged wire; the axis of a thin ring of charge; and the center of a circular arc of charge.
- b. Identify situations in which the direction of the electric field produced by a charge distribution can be deduced from symmetry considerations.



▼ Charge Densities

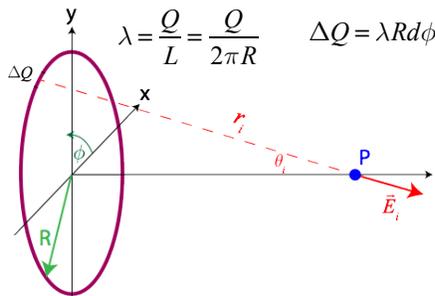
Linear Charge Density $\lambda = \frac{\Delta Q}{\Delta L}$

Surface Charge Density $\sigma = \frac{\Delta Q}{\Delta A}$

Volume Charge Density $\rho = \frac{\Delta Q}{\Delta V}$

▼ Electric Field on the Axis of a Thin Ring of Charge

Find the electric field at a point on the axis (perpendicular to the ring) of a thin insulating ring of radius R that is uniformly charged as shown in the diagram.



$$\lambda = \frac{Q}{L} = \frac{Q}{2\pi R} \quad \Delta Q = \lambda R d\phi$$

▼ Symmetry Arguments

By symmetry, the only net electric field will be in the z-direction.

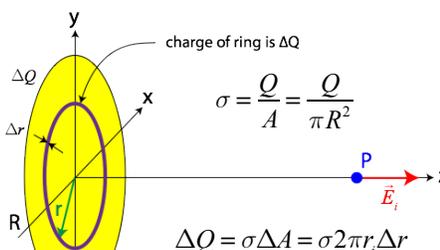
$$\vec{E}_{iz} = E_i \cos \theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i \xrightarrow[r_i = \sqrt{z^2 + R^2}]{\cos \theta_i = \frac{z}{\sqrt{z^2 + R^2}}} E_{iz} = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{(z^2 + R^2)^{3/2}} z \rightarrow$$

$$E_{iz} = \frac{1}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \Delta Q \xrightarrow{\Delta Q = \lambda R d\phi} E_z = \frac{1}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \int_{\phi=0}^{\phi=2\pi} \lambda R d\phi = \frac{1}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \lambda R \int_{\phi=0}^{\phi=2\pi} d\phi \rightarrow$$

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \lambda R (2\pi) \xrightarrow{\lambda = \frac{Q}{2\pi R}} E_z = \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{3/2}}$$

▼ Electric Field Due to a Uniformly Charged Disk

Find the electric field due to a uniformly charged insulating disk of radius R at a point P perpendicular to the disk as shown in the diagram.



$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

$$\Delta Q = \sigma \Delta A = \sigma 2\pi r_i \Delta r$$

▼ Symmetry Arguments

By symmetry, the only net electric field will be in the z-direction.

$$E_{iz} = \frac{1}{4\pi\epsilon_0} \frac{z \Delta Q_i}{(z^2 + r_i^2)^{3/2}} \xrightarrow{\Delta Q = \sigma 2\pi r_i dr} E_z = \int \frac{1}{4\pi\epsilon_0} \frac{z \sigma 2\pi r_i dr}{(z^2 + r_i^2)^{3/2}} \rightarrow$$

$$E_z = \frac{\sigma z}{2\epsilon_0} \int_{r=0}^R \frac{r dr}{(z^2 + r_i^2)^{3/2}} \xrightarrow[u=z^2+r^2]{du=2r dr} E_z = \frac{\sigma z}{2\epsilon_0} \frac{1}{2} \int_{u=z^2}^{u=z^2+R^2} \frac{du}{u^{3/2}} = \frac{\sigma z}{4\epsilon_0} \left(\frac{-2}{u^{1/2}} \right) \Big|_{z^2}^{z^2+R^2} \rightarrow$$

$$E_z = \frac{\sigma z}{4\epsilon_0} \left(\frac{-2}{\sqrt{z^2 + R^2}} - \frac{-2}{z} \right) = \frac{\sigma z}{2\epsilon_0} \left(\frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}} \right) \rightarrow E_z = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

▼ What is the E field if the disc is infinite (an infinite plane)?

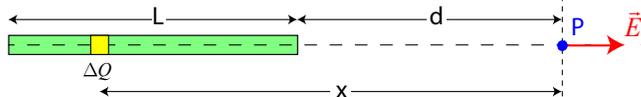
$$\lim_{R \rightarrow \infty} \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) = \frac{\sigma}{2\epsilon_0}$$

▼ Electric Field due to a Finite Charged Rod

Find the electric field due to a finite uniformly charged rod of length L lying on its side at some distance d away from the end of the rod.

▼ Symmetry Arguments

By symmetry, the only electric field will be in the x-direction.



$$\lambda = \frac{Q}{L} \rightarrow \Delta Q = \lambda \Delta x \rightarrow dQ = \lambda dx$$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{x^2} \rightarrow E_x = \int \frac{1}{4\pi\epsilon_0} \frac{dQ}{x^2} \xrightarrow{dQ = \lambda dx} E_x = \int_{x=d}^{x=d+L} \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{x^2} = \frac{\lambda}{4\pi\epsilon_0} \int_d^{d+L} \frac{dx}{x^2} \rightarrow E_x = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{-1}{x} \right) \Big|_d^{d+L} \rightarrow$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{-1}{d+L} - \frac{-1}{d} \right) = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{-d + d + L}{d(d+L)} \right) = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{L}{d(d+L)} \right) \xrightarrow{\lambda = Q/L} E_x = \frac{Q}{4\pi\epsilon_0 L} \frac{L}{d(d+L)} \rightarrow E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{d(d+L)}$$

Gauss's Law

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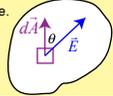
1. Gauss's Law

- ▼ a. Understand the relationship between electric field and electric flux.
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 - i. State Gauss's Law in integral form and apply it qualitatively to relate flux and electric charge for a specified surface.
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▼ Electric Flux Through Open Surfaces

Electric Flux (Φ) is the amount of electric field penetrating a surface.

$$d\Phi = \vec{E} \cdot d\vec{A} = EdA \cos \theta \rightarrow \Phi = \int d\Phi = \int_A \vec{E} \cdot d\vec{A}$$

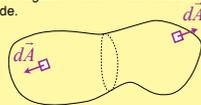


▼ Electric Flux Through Closed Surfaces

Convention: Normals to closed surfaces point from the inside to the outside. Total flux through the closed surface is positive if there is more flux from inside to outside than outside to inside, and negative if there is more flux from outside to inside than inside to outside.

$$\Phi = \int d\Phi = \oint \vec{E} \cdot d\vec{A}$$

Integral over closed surface



▼ Derivation of Gauss's Law

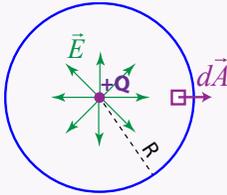
Consider a point charge inside a spherical shell of radius R. Determine the flux through the sphere.

$$d\Phi = \vec{E} \cdot d\vec{A} = EdA \cos \theta \xrightarrow{\theta=0} \xrightarrow{\cos \theta=1} d\Phi = EdA \rightarrow$$

$$\Phi = \int d\Phi = \oint E dA = E \oint dA = EA \xrightarrow{A=4\pi R^2} \rightarrow$$

$$\Phi = 4\pi R^2 E \xrightarrow{E = \frac{Q}{4\pi\epsilon_0 R^2}} \rightarrow \Phi = 4\pi R^2 \left(\frac{Q}{4\pi\epsilon_0 R^2} \right) = \frac{Q}{\epsilon_0} \rightarrow$$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0} \leftarrow \text{Gauss's Law!}$$



▼ Gauss's Law

Useful for finding the electric field due to charge distributions for cases of:

- 1) Spherical symmetry
- 2) Cylindrical symmetry
- 3) Planar symmetry

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

▼ Sample Problem: Electric Field due to a Thin Hollow Shell

Consider a thin hollow shell of uniformly distributed charge Q. Find the electric field inside and outside the sphere.

Choose a "Gaussian Surface" as a sphere (first inside the shell of charge, then outside the shell of charge).

By symmetry, the electric field at all points on the Gaussian spheres must be the same, and it must point radially in or out.

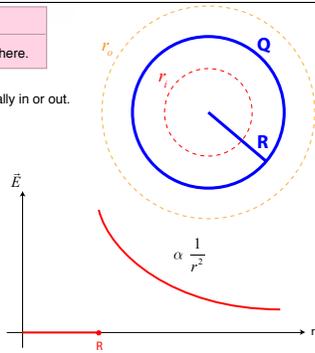
Inside the shell of charge ($r < R$):

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0} \rightarrow 4\pi r^2 E = \frac{Q_{enc}}{\epsilon_0} \rightarrow E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} \xrightarrow{Q_{enc}=0} \rightarrow E = 0$$

Outside the shell of charge ($r > R$):

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0} \rightarrow 4\pi r^2 E = \frac{Q_{enc}}{\epsilon_0} \rightarrow E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2}$$

Same answer as if all the charge Q was placed at a point in the center of the sphere.



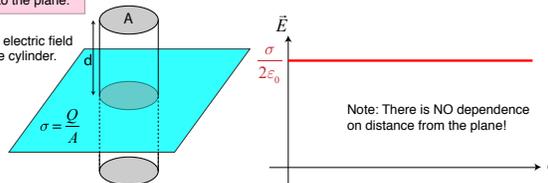
▼ Sample Problem: Electric Field due to an Infinite Plane

Consider an infinite plane of uniform charge density σ . Determine the electric field due to the plane.

Choose a "Gaussian Surface" as a cylinder as shown in the diagram. By symmetry, the electric field at all points on the cylinder must point perpendicular to the plane through the caps of the cylinder.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow{Q_{enc} = \sigma A} \Phi_{E_{top}} + \Phi_{E_{bottom}} + \Phi_{E_{sides}} = \frac{\sigma A}{\epsilon_0} \xrightarrow{\text{symmetry}} \xrightarrow{\Phi_{E_{sides}} = 0}$$

$$\Phi_{E_{top}} + \Phi_{E_{bottom}} = \frac{\sigma A}{\epsilon_0} \xrightarrow{\Phi_{E_{top}} = EA} \xrightarrow{\Phi_{E_{bottom}} = EA} \rightarrow 2EA = \frac{\sigma A}{\epsilon_0} \rightarrow E = \frac{\sigma}{2\epsilon_0}$$



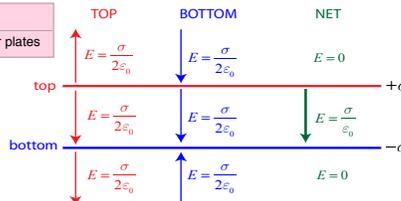
Note: There is NO dependence on distance from the plane!

▼ Sample Problem: Electric Field Between Parallel Charged Planes

Find the electric field surrounding and in between two oppositely-charged parallel planes or plates

Strategy: Use E Field from a single infinite plane with surface charge density σ to derive solution by adding the electric fields from each of the planes using the superposition principle.

Note that this is not accurate near the ends of the planes or plates.



▼ Sample Problem: Electric Field due to Infinite Line of Charge

Determine the electric field at a distance R from an infinitely long line of uniform charge density λ .

Choose a "Gaussian Surface" as a cylinder centered around the line of charge as shown. By symmetry, the electric field at all points on the cylinder must point radially in or out.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \rightarrow \Phi_L + \Phi_R + \Phi_{Cyl} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow{\text{symmetry}} \xrightarrow{\Phi_L + \Phi_R = 0} \Phi_{Cyl} = \frac{Q_{enc}}{\epsilon_0} \xrightarrow{\Phi_{Cyl} = 2\pi RLE} \rightarrow$$

$$2\pi RLE = \frac{Q_{enc}}{\epsilon_0} \xrightarrow{Q_{enc} = \lambda L} \rightarrow 2\pi RLE = \frac{\lambda L}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 R}$$

Same answer as that reached using Coulomb's Law

